Otto von Guericke Universität Magdeburg **Summer Term 2016**
Fakultät II – Mathematik und Naturwissenschaften
Institut für Mathematik
Jan Heiland as of: May 9, 2016

Differential Algebraic Equations

Exercise Sheet 1 – Linear DAEs with constant coefficients

A Regularity and Kronecker Forms
Check whether the matrix pairs

$$\left(\begin{bmatrix}1 & 1 & 0\\ 0 & -1 & 1\\ 0 & 0 & 0\end{bmatrix}, \begin{bmatrix}1 & 0 & 0\\ 0 & 1 & -1\\ 0 & -1 & 1\end{bmatrix}\right) \quad \text{and} \quad \left(\begin{bmatrix}2 & -1 & 1\\ 3 & -2 & 2\\ 0 & 0 & 0\end{bmatrix}, \begin{bmatrix}1 & 0 & 0\\ 0 & 1 & -1\\ 1 & -1 & 1\end{bmatrix}\right)$$

are regular or singular and determine their Kronecker canonical forms by elementary row and column transforms.

B Index-1 condition
Show that the matrix pair

$$(E, A) = \left(\begin{bmatrix}I_r & 0\\ 0 & 0\end{bmatrix}, \begin{bmatrix}A_{11} & A_{12}\\ A_{21} & A_{22}\end{bmatrix}\right)$$

with E, $A \in \mathbb{C}^{n,n}$, and $r < n$, is of index 1 if, and only if, A_{22} is square and nonsingular.

C Regularity and commutativity
Let E, $A \in \mathbb{C}^{n,n}$ satisfy $EA = AE$. Show

1. that (E, A) is regular if, and only if, $\operatorname{kernel} E \cap \operatorname{kernel} A = \{0\}$
2. and that $\operatorname{ind}(E, A) = \operatorname{ind} E$.

D Regularity and commutativity II
Let (E, A) be regular with E, $A \in \mathbb{C}^{n,n}$. For a $\tilde{\lambda}$ such that $\tilde{\lambda}E - A$ is invertible, show

1. that $\tilde{E} := (\tilde{\lambda}E - A)^{-1}E$ and $\tilde{A} := (\tilde{\lambda}E - A)^{-1}A$ commute
2. and that $\operatorname{ind}(E, A) = \operatorname{ind} \tilde{E}$.

E Drazin inverse as group inverse
If $\operatorname{ind} E \leq 1$, then the Drazin inverse E^D is also called group inverse of E and denoted by $E^\#$. Show that $E \in \mathbb{C}^{n,n}$ is an element of *a* group $\mathbb{G} \subset \mathbb{C}^{n,n}$ with the matrix multiplication if and only if $\operatorname{ind} E \leq 1$, and that the inverse in such a group is just $E^\#$. (Note: The question is whether E is a member of some group. I was wrong in the lecture. The Drazin inverse does not extend the group of regular matrices to matrices of index smaller or equal 1)

F Drazin inverse property
Prove that $((E^D)^D)^D = E^D$ for all $E \in \mathbb{C}^{n,n}$