Otto von Guericke Universität Magdeburg **Summer Term 2018**
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Differential Algebraic Equations

Exercise Sheet 3 – Linear DAEs with Time-varying Coefficients

A (Local) Equivalence Transformation
Prove that the (local) equivalence transformation defined in Definition 4.6 in the lecture defines an equivalence relation.

B (Global) Equivalence and Solvability
Compute $\tilde{E} = PEQ$ and $\tilde{A} = PAQ - PE\dot{Q}$ for

$$E(t) = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, \quad A(t) = \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}, \quad P(t) = \begin{bmatrix} t & 1 \\ 1 & 0 \end{bmatrix}, \quad Q(t) = \begin{bmatrix} -1 & t \\ 0 & -1 \end{bmatrix},$$

and for

$$E(t) = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}, \quad A(t) = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \quad P(t) = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}, \quad Q(t) = \begin{bmatrix} 0 & -1 \\ 1 & -t \end{bmatrix},$$

compare it to the initial examples of Section 4 of the lecture, and interpret your observations.

C Characteristic Quantities I
Determine the (local) characteristic quantities (r, a, s) of

$$(E(t), A(t)) = \left(\begin{bmatrix} 0 & 0 \\ 1 & \eta t \end{bmatrix}, \begin{bmatrix} -1 & -\eta t \\ 0 & -(1+\eta) \end{bmatrix} \right) \tag{1}$$

for every $t \in \mathbb{R}$ and for every $\eta \in \mathbb{R}$.

D Characteristic Quantities II
Determine the (local) characteristic quantities (r, a, s) of

$$(E(t), A(t)) = \left(\begin{bmatrix} 0 & 0 \\ 0 & t \end{bmatrix}, \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \right) \tag{2}$$

for every $t \in \mathbb{R}$.

E Drazin Inverse
For $E \in \mathcal{C}(I, \mathbb{R}^{n,n})$, we define the Drazin inverse pointwise E^D pointwise via $E^D(t) = E(t)^D$. Determine the E^D for the matrix functions E from (1) and (2). What do you observe?

F Global Characteristic Quantities
Compute the (global) characteristic quantities (r_i, a_i, s_i), $i = 1, \ldots, \mu$, of the pair of matrix functions (E, A) given in (1).