MAX PLANCK INSTITUTE
FOR DYNAMICS OF COMPLEX
TECHNICAL SYSTEMS
MAGDEBURG

CSC
COMPUTATIONAL METHODS IN
SYSTEMS AND CONTROL THEORY

Implicit and Explicit Matching of Non-Proper Transfer Functions in the Loewner Framework

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joint work with Ion Victor Gosea

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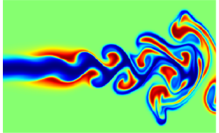
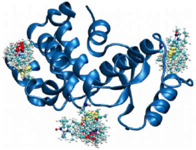
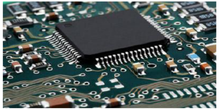
Outline

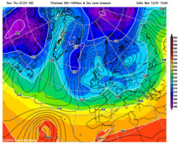
Reduced Order Modelling in Frequency Domain

- **Model Order Reduction** (**MOR**) is used to transform large, complex models of time-dependent processes into smaller, simpler models that are still capable of representing accurately the behavior of the original process.









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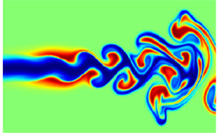

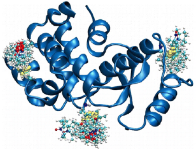

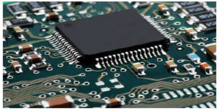

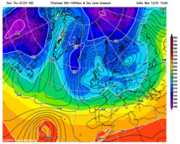

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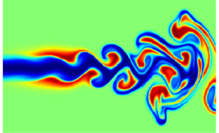
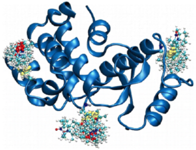
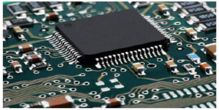
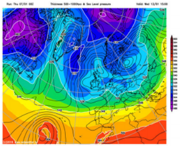
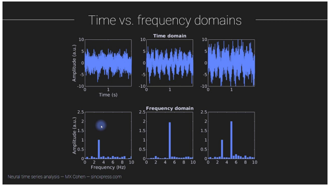





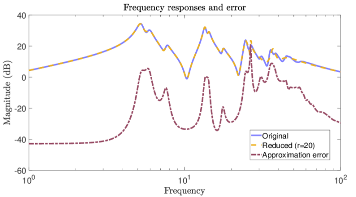
- Data-driven reduced-order modeling (DD-ROM) and learning methods have become more and more appealing and sought after over the years.
- At MPI Magdeburg, we study and develop different DD-ROM + learning methods:

  $\longrightarrow$ Operator Inference (OpInf), Loewner Framework, AAA, DMD, or QuadBT

  $\longrightarrow$ Typical learning methods (with ANNs: LQResNet, CNNs, RNNs), SINDy, . . .

Reduced Order Modelling in Frequency Domain





- In electronics or control systems engineering, the frequency domain refers to the analysis of mathematical functions or signals with respect to frequency.
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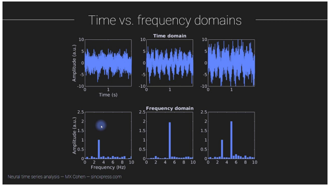

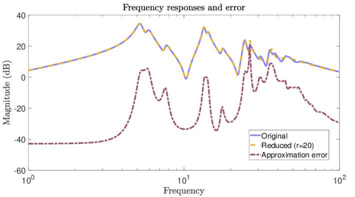

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$$\begin{cases} \mathcal{L}\left[\mathbf{E}\dot{\mathbf{x}}(t)\right] = \mathcal{L}\left[\mathbf{A}\mathbf{x}(t)\right] + \mathcal{L}\left[\mathbf{B}\mathbf{u}(t)\right], \\ \mathcal{L}\left[\mathbf{y}(t)\right] = \mathcal{L}\left[\mathbf{C}\mathbf{x}(t)\right] + \mathcal{L}\left[\mathbf{D}\mathbf{u}(t)\right], \end{cases} \Rightarrow \begin{cases} s\mathbf{E}\hat{\mathbf{x}}(s) - \mathbf{x}_0 = \mathbf{A}\hat{\mathbf{x}}(s) + \mathbf{B}\hat{\mathbf{u}}(s), \\ \hat{\mathbf{y}}(s) = \mathbf{C}\hat{\mathbf{x}}(s) + \mathbf{D}\hat{\mathbf{u}}(s). \end{cases}$$

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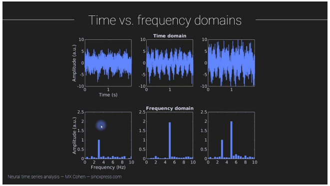

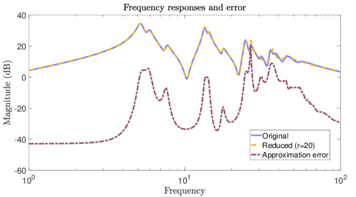

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- By solving the algebraic equation above in terms $\hat{\mathbf{x}}(s)$, assuming $\mathbf{x}_0 = 0$, we get that:

$$\hat{\mathbf{y}}(s) = \underbrace{\left[\mathbf{C}(s\mathbf{E} - \mathbf{A})^{-1}\mathbf{B} + \mathbf{D}\right]}_{\text{the transfer function } \mathbf{H}(s)} \hat{\mathbf{u}}(s). \tag{1}$$

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- Use available measurements and apply data-driven approaches:
  1. Vector fitting - [Gustavsen/Semlyen '99]; [Drmac/Gugercin/Beattie '15];
  2. The RKFIT algorithm - [Berljafa/Güttel '17] (RK toolbox);
  3. The AAA algorithm - [Nakatsukasa/Sete/Trefethen '18] (Chebfun toolbox);
  4. The Loewner framework - [Mayo/Antoulas '07]; [Antoulas/Lefteriu/Ionita '17] . . .

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- Use data samples to construct an approximated fitting model:
  1. Given by matrices $\mathbf{E}, \mathbf{A}, \mathbf{B}, \mathbf{C}$ and TF: $\mathbf{H}(s) = \mathbf{C}(s\mathbf{E} - \mathbf{A})^{-1}\mathbf{B}$.
  2. Given in barycentric representation: $\mathbf{H}(s) = \frac{\sum_{k=0}^{d} \frac{w_k f_k}{s-\xi_k}}{\sum_{k=0}^{d} \frac{w_k}{s-\xi_k}}$.
  3. Given in pole-residue representation: $\mathbf{H}(s) = \eta_0 + \sum_{k=1}^{d} \frac{\beta_k}{s-\xi_k}$.

Rational Interpolation and the Loewner Matrix

- **Lagrange basis** for the linear space of polynomials of degree at most n.

  Given $\lambda_i \in \mathbb{C}$, $i = 1, \cdots, n+1$: $\lambda_i \neq \lambda_j$, $i \neq j$,

$$\mathbf{q}_i(s) := \Pi_{i' \neq i}(s - \lambda_{i'}), \; i = 1, \cdots, n+1,$$

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- The **barycentric Lagrange interpolation** for a rational function $g(s)$ satisfying the (right) interpolation conditions for any $1 \leq i \leq n+1$

$$\mathbf{g}(\lambda_i) = \mathbf{z}_i,$$

  is given by:

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- The free parameters (weights) w_i are found so that *additional* interpolation conditions hold:

$$\mathbf{g}(\mu_j) = \mathbf{v}_j, \ j = 1, \cdots, r,$$

  where $(\mu_j, \mathbf{v}_j)$, with $\mu_i \neq \mu_j$, are given.

J. P. Berrut and N. Trefethen, *Barycentric Lagrange Interpolation*, SIAM Review, 2004.

Rational Interpolation and the Loewner Matrix

- For these extra conditions to be satisfied, one needs to enforce $\mathbb{L}\,\mathbf{c} = 0$, where

$$\mathbb{L} = \begin{bmatrix} \frac{\mathbf{v}_1 - \mathbf{z}_1}{\mu_1 - \lambda_1} & \cdots & \frac{\mathbf{v}_1 - \mathbf{z}_{n+1}}{\mu_1 - \lambda_{n+1}} \\ \vdots & \ddots & \vdots \\ \frac{\mathbf{v}_r - \mathbf{z}_1}{\mu_r - \lambda_1} & \cdots & \frac{\mathbf{v}_r - \mathbf{z}_{n+1}}{\mu_r - \lambda_{n+1}} \end{bmatrix} \in \mathbb{C}^{r\times(n+1)}, \quad \mathbf{c} = \begin{bmatrix} w_1 \\ \vdots \\ w_{n+1} \end{bmatrix} \in \mathbb{C}^{n+1}.$$

- Here, $\mathbb{L}$ is a **Loewner matrix** (from Charles Loewner) with:

  left (row) array $(\mu_j, \mathbf{v}_j)$, $j = 1, \ldots, r$, and **right (column) array** $(\lambda_i, \mathbf{z}_i)$, $i = 1, \ldots, n+1$.

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Main property

Let $\mathbb{L}$ be a $p \times k$ Loewner matrix. Then the following holds:

$$\mathbf{p}, \mathbf{k} \geq \deg(\mathbf{g}) \quad \Rightarrow \quad \operatorname{rank} \mathbb{L} = \deg(\mathbf{g}).$$

Consequently, every square Loewner matrix of size $\deg(\mathbf{g})$, is non-singular.

A.C. Antoulas and B.D.O. Anderson, On the scalar rational interpolation problem, IMA Journal of Mathematical Control and Information, 3: 61–88, 1986.

A Toy Example

- Let $\mathbf{f}(s) = (s^2 + 4)/(s + 1)$ be a rational function of complexity $n := \deg(\mathbf{f}) = 2$.
- By evaluating $(\mathbf{s})$ on $\lambda = [1, 3, 5]$ and $\mu = [2, 4, 6, 8]$, one obtains $\mathbf{z} = [5/2, 13/4, 29/6]$ and $\mathbf{v} = [8/3, 4, 40/7, 68/9]$.

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- Then, we construct the Loewner matrix, its null space (rank$(\mathbb{L}) = 2$), and a rational function interpolating the data as,

$$\mathbb{L} = \begin{bmatrix} \frac{1}{6} & \frac{7}{12} & \frac{13}{18} \\ \frac{1}{2} & \frac{3}{4} & \frac{5}{6} \\ \frac{9}{14} & \frac{23}{28} & \frac{37}{42} \\ \frac{13}{18} & \frac{31}{36} & \frac{49}{54} \end{bmatrix}, \mathbf{c} = \begin{bmatrix} \frac{1}{3} \\ -\frac{4}{3} \\ 1 \end{bmatrix}, \mathbf{g}(s) = \frac{\frac{5}{6\,(s-1)} - \frac{13}{3\,(s-3)} + \frac{29}{6\,(s-5)}}{\frac{1}{3\,(s-1)} - \frac{4}{3\,(s-3)} + \frac{1}{s-5}}.$$

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- In this case, $\mathbf{g}(s)$ perfectly recovers the original function $\mathbf{f}(s)$, i.e., $\mathbf{g}(s) = \mathbf{f}(s)$.
- A matrix-format realization can be obtained as $\hat{\mathbf{H}}(s) = \mathbf{W}\mathbf{\Phi}(s)^{-1}\mathbf{G}$, where

$$\mathbf{\Phi}(s) = \left[\begin{array}{ccc} s-1 & 3-s & 0 \\ s-1 & 0 & 5-s \\ \hline -\frac{1}{3} & \frac{4}{3} & -1 \end{array}\right] \text{ and } \begin{cases} \mathbf{W} = \left[\begin{array}{cc|c} 0 & 0 & -1 \end{array}\right], \\ \mathbf{G}^\top = \begin{bmatrix} \frac{5}{6} & -\frac{13}{3} & \frac{29}{6} \end{bmatrix}. \end{cases}$$

The AAA algorithm - a summary

- The **AAA** algorithm was introduced in [Nakatsukasa/Sete/Trefethen '18] .
- It stands for "*Adaptive Antoulas-Anderson*" in honor of the authors who introduced this type of interpolation scheme in the 80s.

• ***A.C. Antoulas** and **B.D.O. Anderson**,*
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The main steps of the **AAA** algorithm are:

1. Write down rational approximants in a "barycentric" representation.
2. Select the interpolation points ("support points") via a Greedy scheme.
3. Compute the other variables ("weights") to enforce **least squares** approximation.

$\rightsquigarrow$ The block-**AAA** algorithm was developed in [Gosea/Güttel '21] ;
$\rightsquigarrow$ The set-valued **AAA** algorithm was proposed in [Lietaert et al. '22] ;
$\rightsquigarrow$ The **AA** approach was extended to the DAE case (index 2 / relative degree 1) [Gosea/H. '24] – this talk ;
$\rightsquigarrow$ Ongoing work for extending **AAA** to generic DAE cases (index-aware approach) [Pradovera/Gosea/H. '24] – upcoming ;

Why matching polynomial terms?

- We saw that for the toy example:

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- Also, this function has a polynomial part of degree $n = 1$ (meaning that $f(s) = \mathcal{O}(s)$ for $|s| \to \infty$)
- Nonetheless, in practical (more complex) examples, the classical methods fail to accurately reproduce the behavior at high frequencies:

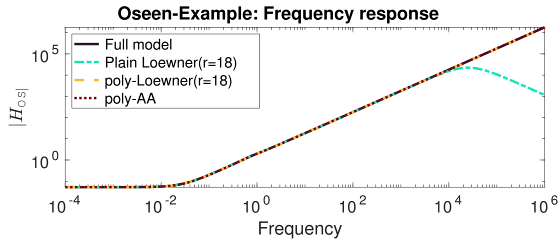

Figure: A typical frequency response plot for systems with polynomial parts...

- In [Gosea/H. '24] the classical barycentric form is modified to account for the case of higher-index DAEs, i.e., with index $\nu = 2$, as simple as:

$$\boxed{\tilde{\mathbf{g}}(s) = \frac{q + \displaystyle\sum_{i=1}^{n+1} \frac{w_i \mathbf{z}_i}{s - \lambda_i}}{\displaystyle\sum_{i=1}^{n+1} \frac{w_i}{s - \lambda_i}}}$$

- The free parameters (weights) w_i + the coefficient q can be also found as before, i.e., so that additional interpolation conditions hold:

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- To do so, we need to solve the following equation:

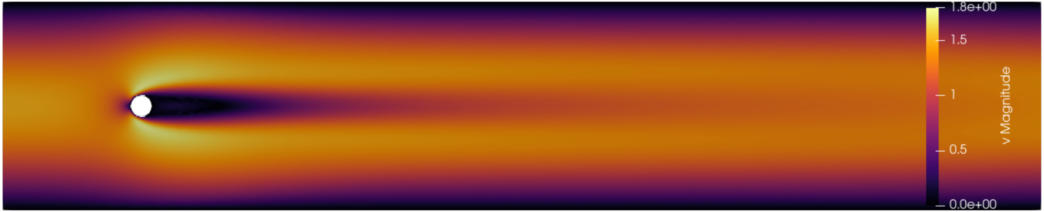
$$\tilde{\mathbb{L}}\tilde{\mathbf{c}} = \mathbf{0},$$

where the augmented Loewner matrix is written as:

$$\tilde{\mathbb{L}} = \begin{bmatrix} \mathbb{L} & -\mathbf{1}_{n+1} \end{bmatrix}, \text{ and } \tilde{\mathbf{c}} = \begin{bmatrix} \mathbf{c} \\ q \end{bmatrix}.$$

Numerical Example

- We consider the flow past a cylinder in 2 dimensions
- at Reynolds number 20 calculated with the averaged inflow velocity and the cylinder diameter as reference quantities;
- see [Gosea/H. '24] adapted from [Ahmad et al. '17] .



Snapshot of magnitude of the steady-state NS velocity solution in the considered setup.

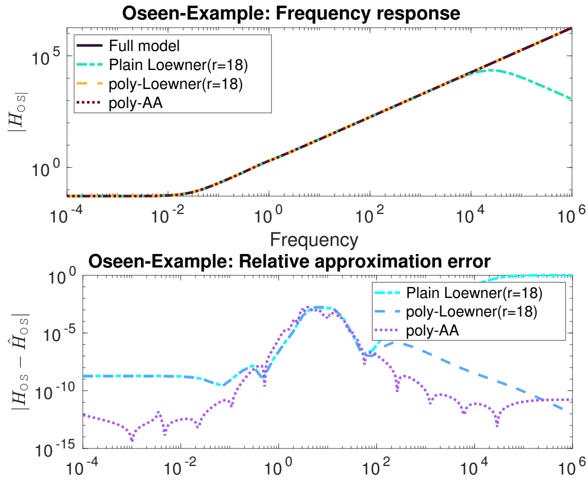
- The considered flow problem with boundary control is modeled by a finite element discretization of the incompressible Oseen equations.
- The Oseen equations are obtained from the Navier-Stokes equations by a Newton linearization about a steady state solution.
- The control $\nu(t,x)$ distributed over the boundary, is modeled as $\nu(t,x) = g(x)u(t)$ through a function $g\colon \Gamma \to \mathbb{R}^2$ that describes the spatial extension.
- Overall, the spatially-discretized model for the velocity v and pressure p reads

$$\begin{aligned} \begin{bmatrix} M & M_\Gamma \end{bmatrix} \begin{bmatrix} \dot{v}(t) \\ \dot{v}_\Gamma(t) \end{bmatrix} &= \begin{bmatrix} A & A_\Gamma \end{bmatrix} \begin{bmatrix} v(t) \\ v_\Gamma(t) \end{bmatrix} + J^T p(t), \\ 0 &= \begin{bmatrix} J & J_\Gamma \end{bmatrix} \begin{bmatrix} v(t) \\ v_\Gamma(t) \end{bmatrix}, \quad 0 = v_\Gamma(t) - b_\Gamma u(t), \\ y(t) &= C_v v(t) + C_p p(t). \end{aligned} \tag{2}$$

- The transfer function when considering the y_p output only, with $\mathcal{C}_p = \begin{bmatrix} 0 & C_p \end{bmatrix}$, is:

$$H_{\mathrm{OS}}(s) := \mathcal{C}_p (s\mathcal{E} - \mathcal{A})^{-1} (\mathcal{B}_1 + s\mathcal{B}_2). \tag{3}$$

We compare with the classical (plain) Loewner framework (LF) [Mayo/Antoulas '07] , and with the post processing LF method in [Antoulas/Gosea/Heinkenschloss '20] .



Summary and Conclusion

- Proposed a variant of Loewner-based system identification with free parameters in the Antoulas-Anderson algorithm
- that implicitly covers polynomial parts of the transfer function, avoiding the need for high-frequency data points.
- Drawback: Reduced error control on coefficients, leading to larger approximation errors at high frequencies.
- Future work: adaptive algorithms, like the adaptive Antoulas-Anderson approach.
- Ongoing work: extending the approach to higher polynomial terms and automatic detection of the polynomial degree.

Thank you!

psst... PhD wanted $\rightarrow$

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